

# A Multivariate Model of Strategic Asset Allocation with Longevity Risk: Online Appendix

Emilio Bisetti, Carlo A. Favero, Giacomo Nocera, Claudio Tebaldi

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*Email addresses:* `bisetti@cmu.edu` (Emilio Bisetti), `carlo.favero@unibocconi.it` (Carlo A. Favero), `gnocera@audencia.com` (Giacomo Nocera), `claudio.tebaldi@unibocconi.it` (Claudio Tebaldi)

## Appendix A. Identification and estimation of the Lee-Carter mortality model

The Lee and Carter (1992) model consists of a system of equations for logarithms of central death rates for age cohort  $x$  at time  $t$ ,  $m_{x,t}$ , and a time-series equation for an unobservable time-varying mortality index  $k_t$ :

$$(A.1) \quad \ln(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t}$$

$$(A.2) \quad k_t = c_0 + c_1 k_{t-1} + e_t$$

$$\epsilon_{x,t} \sim \text{NID}(0, \sigma_\epsilon^2)$$

$$e_t \sim \text{MeanZero - Stationary Process}$$

where  $a_x$  and  $b_x$  are age-specific constants. The error term  $\epsilon_{x,t}$  captures cross-sectional errors in the model based prediction for mortality of different cohorts, while the error term  $e_t$  captures random fluctuations in the time series of the common factor  $k_t$  driving mortality at all ages. This common factor, usually known as the unobservable mortality index evolves over time as an autoregressive process and the favorite Carter-Lee specification makes is a unit-root process by setting  $c_1 = 1$ . Identification is achieved by imposing the restrictions  $\sum_t k_t = 0$  and  $\sum_x b_x = 1$ , so that the unobserved mortality index  $k_t$  is estimated through Singular Value Decomposition (SVD). SVD is a technique based on a theorem of linear algebra stating that a  $(m \times n)$  rectangular matrix  $M$  can be broken down into the product of three matrices - an  $(m \times m)$  orthogonal matrix  $U$ , a diagonal  $(m \times n)$  matrix  $S$ , and the transpose of an orthogonal  $(n \times n)$  matrix  $V$ . The SVD of the matrix  $M$  will be therefore be given by  $M = USV'$  where  $U'U = I$  and  $V'V = I$ . The columns of  $U$  are

orthonormal eigenvectors of  $AA'$ , the columns of  $V$  are orthonormal eigenvectors of  $A'A$ , and  $S$  is a diagonal matrix whose elements are the square roots of eigenvalues from  $U$  or  $V$  in descending order. The restriction  $\sum_t k_t = 0$  implies that  $a_x$  is the average across time of  $q_{x,t}$ , and Equation A.1 can be rewritten in terms of the mean-centered log-mortality rate as

$$(A.3) \quad m_{x,t} - \bar{m}_{x,t} \equiv \tilde{m}_{x,t} = b_x k_t + \epsilon_{x,t}.$$

Grouping all the  $\tilde{m}_{x,t}$  in a unique  $(X \times T)$  matrix  $\tilde{m}$  (where the columns are mortality rates at time- $t$  ordered by age groups and the rows are mortality rates through time for a specific age-group  $x$ ), leads naturally to use SVD to obtain estimates of  $b_x$  and  $k_t$ . In particular, if  $\tilde{m}$  can be decomposed as  $\tilde{m} = USV'$ ,  $b = [b_0, b_1, \dots, b_X]$  is represented by the normalized first column of  $U$ ,  $u_1 = [u_{0,1}, u_{1,1}, \dots, u_{X,1}]$ , so that

$$b = \frac{u_1}{\sum_{x=0}^X u_{x,1}}.$$

On the other hand the mortality index vector  $k = [k_1, k_2, \dots, k_T]$  is given by

$$k = \lambda_1 \left( \sum_{x=0}^X u_{x,1} \right) \nu_1$$

where  $\nu_1 = [\nu_{1,1}, \nu_{1,2}, \dots, \nu_{1,T}]'$  is the first column of the  $V$  matrix and  $\lambda_1$  is the highest eigenvalue of the matrix  $S$ . The values of mortality rates obtained with this method will not, in general, be equal to the actual number of deaths. In Lee and Carter (1992), the authors hence re-estimate  $k_t$  in a second step, taking the values of  $a_x$  and  $b_x$  as given from

the first-step SVD estimate and using actual mortality rates. The new values of  $k$  are obtained so that, for each year, the actual death rates are equal to the implied ones. This two-step procedure allows to take into account the population age distribution, providing a very good fit for 13 of the 19 age groups in the authors' sample, where more than 95% of the variance over time is explained. For seven of these, more than 98% of the variance is explained.

In Figure A.1 we report the performance of the Lee-Carter model in fitting the US mortality rates. Our data come from the Human Mortality Database of the University of Berkeley.<sup>1</sup> In Figure A.1.1 we plot realized mortality at age 65 throughout the period 1952-2010 (red dashed line) against its Lee-Carter fitted value (blue continuous line). In Figure A.1.2, we report the cross-sectional  $R^2$  of the estimate for all age cohorts in the same period. The model performs very well in fitting mortality rates at all ages but those greater than 95, where the volatility of mortality is high: for more than fifty percent of ages, the  $R^2$  is above 95%, and for more than seventy-five percent of ages it is above 80%. Figure A.1.3 reports the estimated unobservable common mortality index  $k$  from Equation (A.2), which, given the "four hours a day" evidence, clearly features a negative trend. The autoregressive coefficient  $c_1$  from Equation (A.2) is not statistically different from one and we therefore restrict it to a unit value. Figure A.1.4 reports the innovations in the unobservable mortality index

**Insert Figure A.1 about here**

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<sup>1</sup>The data are publicly available at <http://www.mortality.org/>

**Appendix B. Financial asset returns and their predictors: the basic specification of the VAR model for traditional financial investments**

Consider the continuously compounded security market returns from time  $t$  to time  $t + 1$ ,  $\mathbf{r}_{t+1}$ . Define  $\mu_t$ , the conditional expected log return given information up to time  $t$  as follows:

$$\mathbf{r}_{t+1} = \mu + \mathbf{u}_{t+1},$$

where  $\mathbf{u}_{t+1}$  is the unexpected log return. Define the  $\tau$ -period cumulative return from period  $t + 1$  through period  $t + \tau$ , as

$$\mathbf{r}_{t,t+\tau} = \sum_{i=1}^{\tau} \mathbf{r}_{t+i}.$$

The term structure of risk is defined as the conditional variance of cumulative returns, given the investor's information set, scaled by the investment horizon

$$(B.1) \quad \Sigma_r(\tau) \equiv \frac{1}{\tau} \text{Var}(\mathbf{r}_{t,t+\tau} \mid D_t),$$

where  $D_t^{\text{Mkt}} \equiv \sigma\{z_\tau^{\text{Mkt}} : \tau \leq t\}$  consists of the full histories of returns as well as predictors that investors use in forecasting returns. Following Barberis (2000) and Campbell and Viceira (2002), we describe asset return dynamics by means of a first-order vector autoregressive or VAR(1) model. We choose a VAR(1) as the inclusion of additional lags, even if easily implemented, would reduce the precision of the estimates:

$$(B.2) \quad \mathbf{z}_t^{\text{Mkt}} = \Phi_0^{\text{Mkt}} + \Phi_1^{\text{Mkt}} \mathbf{z}_{t-1}^{\text{Mkt}} + \nu_t^{\text{Mkt}},$$

where

$$\mathbf{z}_t^{\text{Mkt}} = \begin{bmatrix} r_{0t} \\ \mathbf{x}_t^{\text{Mkt}} \\ \mathbf{s}_t^{\text{Mkt}} \end{bmatrix}$$

is a  $(m \times 1)$  vector, with  $r_{0t}$  being the log real return on the asset used as a benchmark to compute excess returns on all other asset classes,  $\mathbf{x}_t^{\text{Mkt}}$  being the  $(n \times 1)$  vector of log excess returns on all other asset classes with respects to the benchmark, and  $\mathbf{s}_t^{\text{Mkt}}$  is the  $((m - n - 1) \times 1)$  vector of returns predictors. In the VAR(1) specification,  $\Phi_0^{\text{Mkt}}$  is a  $(m \times 1)$  vector of intercepts and  $\Phi_1^{\text{Mkt}}$  is a  $(m \times m)$  matrix of slopes. Finally,  $\nu_t^{\text{Mkt}}$  is a  $(m \times 1)$  vector of innovations in asset returns and returns' predictors for which standard assumptions apply, i.e.

$$(B.3) \quad \nu_t^{\text{Mkt}} \sim \mathcal{N}(0, \Sigma_\nu^{\text{Mkt}}),$$

where  $\Sigma_\nu^{\text{Mkt}}$  is the  $(m \times m)$  variance-covariance matrix. Note that

$$\Sigma_\nu^{\text{Mkt}} = \begin{bmatrix} \sigma_0^2 & \sigma'_{0x} & \sigma'_{0s} \\ \sigma_{0x} & \Sigma_{xx} & \Sigma'_{xs} \\ \sigma_{0s} & \Sigma_{xs} & \Sigma_{ss} \end{bmatrix}$$

and the unconditional mean and variances-covariance matrix of  $\mathbf{z}_t$ , assuming that the VAR is stationary and therefore that this moments are well-defined, can be represented as follows:

$$\begin{aligned}\mu_z^{\text{Mkt}} &= (I_m - \Phi_1^{\text{Mkt}})^{-1} \Phi_0^{\text{Mkt}} \\ \text{vec}(\Sigma_{zz}^{\text{Mkt}}) &= (I_{m^2} - \Phi_1^{\text{Mkt}} \otimes \Phi_1^{\text{Mkt}})^{-1} \text{vec}(\Sigma_\nu^{\text{Mkt}}).\end{aligned}$$

The conditional mean of the cumulative asset returns at different horizons are instead

$$E_t(z_{t+1}^{\text{Mkt}} + \dots + z_{t+\tau}^{\text{Mkt}}) = \left( \sum_{i=0}^{\tau-1} (\tau - i) (\Phi_1^{\text{Mkt}})^i \right) \Phi_0^{\text{Mkt}} + \left( \sum_{j=0}^{\tau} (\Phi_1^{\text{Mkt}})^j \right) \mathbf{z}_t^{\text{Mkt}},$$

and their variance is:

$$\begin{aligned}\text{Var}_t(z_{t+1}^{\text{Mkt}} + \dots + z_{t+\tau}^{\text{Mkt}}) &= \Sigma_\nu^{\text{Mkt}} + (I + \Phi_1^{\text{Mkt}}) \Sigma_\nu^{\text{Mkt}} (I + \Phi_1^{\text{Mkt}})' + \\ \text{(B.4)} \quad & (I + \Phi_1^{\text{Mkt}} + (\Phi_1^{\text{Mkt}})^2) \Sigma_\nu^{\text{Mkt}} (I + \Phi_1^{\text{Mkt}} + (\Phi_1^{\text{Mkt}})^2)' + \dots \\ & + (I + \Phi_1^{\text{Mkt}} + \dots + (\Phi_1^{\text{Mkt}})^{\tau-1}) \Sigma_\nu (I + \Phi_1^{\text{Mkt}} + \dots + (\Phi_1^{\text{Mkt}})^{\tau-1})'.\end{aligned}$$

Once the conditional moments of excess returns are available the following selector matrix extracts for each period,  $\tau$ -period conditional moments of log real returns

$$M_r = \begin{bmatrix} 1 & 0_{1 \times n} & 0_{1 \times (m-n-1)} \\ \iota_{n \times 1} & I_{n \times n} & 0_{n \times (m-n-1)} \end{bmatrix}$$

which implies

$$\frac{1}{\tau} \begin{bmatrix} E_t(r_{0,t+1}^\tau) \\ E_t(\mathbf{r}_{t+1}^\tau) \end{bmatrix} = \frac{1}{\tau} M_r E_t(z_{t+1}^{\text{Mkt}} + \dots + z_{t+\tau}^{\text{Mkt}})$$

$$\frac{1}{\tau} \begin{bmatrix} \text{Var}_t (r_{0,t+1}^\tau) \\ \text{Var}_t (\mathbf{r}_{t+1}^\tau) \end{bmatrix} = \frac{1}{\tau} M_r \text{Var}_t (z_{t+1}^{\text{Mkt}} + \dots + z_{t+\tau}^{\text{Mkt}}) M_r'$$

Therefore after the estimation of the VAR it is possible to derive unconditional and conditional moments for returns and excess returns at all different investment horizons. These moments deliver the dynamics of returns and the risk of different assets across investment horizons. This information forms the input for portfolio allocation. Following Campbell and Viceira (2005), we consider a benchmark portfolio to be obtained by attributing optimal weights to bond, stock and T-bills. Therefore we include in  $\mathbf{x}_t^{\text{Mkt}}$  excess returns on stocks and bonds, real returns on T-bills, while we include in  $\mathbf{s}_t^{\text{Mkt}}$  three factors commonly recognized as good predictors of these assets' returns. In particular, the predictors are the nominal short-term interest rate, the dividend price ratio and the yield spread between long-term and short term bonds.

Estimation results are reported in Table B.1. Correlations among financial securities are reported in Figure B.1.

**Insert Table B.1 about here**

**Insert Figure B.1 about here**



A. First order condition for the replication portfolio of  $qk_t$

$$\begin{aligned}
& \min_{\mathbf{w}} \text{Var}_{t-1} \left[ R_t^{qk}(\mathbf{w}) - qk_t \right] \\
&= \min_{\mathbf{w}} \text{Var}_{t-1} [qk_t] + \text{Var}_{t-1} \left[ R_t^{qk}(\mathbf{w}) \right] - 2\text{Cov}_{t-1} \left[ qk_t, R_t^{qk}(\mathbf{w}_{t-1}) \right] \\
&= \min_{\mathbf{w}} \text{Var}_{t-1} \left[ R_t^{qk}(\mathbf{w}) \right] - 2\text{Cov}_{t-1} \left[ qk_t, R_t^{qk}(\mathbf{w}) \right] \\
&= \min_{\mathbf{w}} \text{Var}_{t-1} [\mathbf{w} \cdot \mathbf{x}_t + W_0 r t b_t] - 2\text{Cov}_{t-1} [qk_t, \mathbf{w} \cdot \mathbf{x}_t + W_0 r t b_t] \\
&= \min_{\mathbf{w}} \mathbf{w} \text{Var}_{t-1} [\mathbf{x}_t] \mathbf{w}^T + \mathbf{w} \cdot \{-2\text{Cov}_{t-1} [\mathbf{x}_t, qk_t]\}
\end{aligned}$$

B. Conditional vs Unconditional Sharpe Ratios

$$\begin{aligned}
\text{SR}_{\tau, \mathbf{r}_t} &= \frac{\text{Ex}_{\tau, \mathbf{r}_t}}{\text{Std}_{\tau, \mathbf{r}_t}}, \\
\text{Std}_{\tau, \mathbf{r}_t}(\mathbf{w}) &= \sqrt{\text{Var}_{t, \mathbf{r}_t} \left[ \sum_{k=1}^{\tau} \mathbf{w} \cdot (\mathbf{r}_{t+k} + r_{0,t+k} \mathbf{l}) \right]} / \tau, \\
\text{Std}_{\tau, \mathbf{r}_t}(w_{rtb}) &= \sqrt{\text{Var}_{t, \mathbf{r}_t} \left[ \sum_{k=1}^{\tau} w_{rtb} r_{0,t+k} \right]} / \tau, \\
\text{Ex}_{\tau, \mathbf{r}_t} &= E_{t, \mathbf{r}_t} \left[ \frac{1}{\tau} \sum_{k=1}^{\tau} \mathbf{w} \cdot \mathbf{r}_{t+k} \right] + \frac{\text{Std}_{\tau, \mathbf{r}_t}^2(\mathbf{w})}{2\tau}.
\end{aligned}$$

Figure B.2 reports the term structure of equity Sharpe Ratios obtained by setting the initial condition of the state variables to their long-term expected values.

**Insert Figure B.2 about here**

## Appendix C. The dataset

### A. Annuity prices

We estimate the change in the annuity price included in our VAR(1) model as

$$\Delta p_{65,t+1} = \ln \left( \frac{P_{t+1,65}^A}{P_{t,65}^A} \right),$$

where  $P_{t+1,65}^A$  is the annuity price offered on the US market for 1\$ monthly life annuities written on 65-year-old males. Consequently  $\Delta p_{t+1}^A$  is the yearly log price change of a standardized 65 year-old annuity between time  $t$  and  $t + 1$ . Our annuity prices consist of the sample average premiums for immediate \$1 monthly life annuities for 65-year old US males issued during the 1952-2010 period. In order to have the longest time series of prices, we collected premia from different sources. Following Warshawsky (1988) and Friedman and Warshawsky (1988), premia over the 1952-1967 period come from successive annual issues of Spectator's Handy Guide and A.M. Best's Flitcraft Compend , whereas premia over the 1968-1985 years come from the successive annual issues of A.M. Best's Flitcraft Compend. Following Kojien and Yogo (2012), Cox and Lin (2007) and Brown et al. (2002), we compile premia for the 1986-2010 years from the semiannual issues of Annuity Shopper. These data are integrated with those obtained from the annual issues of the Life/Health editions of Best's Review for the 1995-1998 period.

Given the length of the sample period and the different sources of data we use, our sample premia refer to an unbalanced panel of companies. Although the correct approach should be to use only rates reported by the same companies, this would substantially reduce the number of premia available each year for computing the minimum, the maximum and

the average annual annuity premium which, consequently, might not be reflective of the true value of annuity price. The pricing approach adopted in the present paper is essentially based on the assumption that price changes of annuities reflect changes of fundamental risks priced by buyers and sellers. A side product of this estimation analysis is a direct empirical measure of the effective price reaction to changes in aggregate mortality trends. Clearly, empirical evidence of this connection would support the hypothesis that some form of competition drives the prices in the market for annuities.

### *B. Marketed securities and the state variables*

The Campbell and Viceira (2005) model is developed using quarterly data. As adapting the mortality series to this frequency is both hardly feasible (lack of mortality data for frequencies higher than yearly) and less meaningful (for example, some months of the year experience higher mortality rates than others), we focus our analysis on annual data. We download the financial data from Robert Shiller's website<sup>2</sup> for the postwar period 1952-2010 and, following Campbell et al. (2003) construct the financial time series as:

- Short-term ex-post real rate: return on 6-month commercial paper bought in January and rolled over July, minus the Producer Price Index (PPI).
- Excess return on stocks: log return on the S&P 500 Stocks, from which the short-term interest rate is subtracted.
- Excess return on bonds: returns are obtained using the loglinear approximation de-

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<sup>2</sup><http://www.econ.yale.edu/~shiller/data.htm>

scribed in Campbell et al. (1998)

$$r_{n,t+1} = D_{n,t}y_{n,t} - (D_{n,t} - 1)y_{n-1,t+1},$$

where  $n$  is the Bond maturity, the Bond yield is  $Y_{n,t}$ , the log Bond yield is  $y_{n,t} = \ln(1 + Y_{n,t})$  and  $D_{n,t}$  is the Bond duration, calculated at time  $t$  as

$$D_{n,t} \approx \frac{1 - (1 + Y_{n,t})^{-n}}{1 - (1 + Y_{n,t})^{-1}}$$

with  $n$  set to 20 years and  $y_{n-1,t+1}$  approximated by  $y_{n,t+1}$ .

- Excess annuity prices' growth, as described in the previous subsection of this Appendix.

Finally the following set of state variables are included in the VAR to parametrize the opportunity set faced by the investor

- Nominal T-bill rate: return on 6-month commercial paper bought in January and rolled over July.
- Log dividend-price ratio: natural logarithm of the S&P 500 dividend series minus the logarithm of the S&P 500 price series.
- Yield spread: difference between the log yield of the long Bond and the short yield on the commercial paper.
- Aggregate longevity shocks:  $qk_t$  as defined in Section III, are average differences between predicted and fitted mortality rates for the cohorts underlying life annuities

(in our case, 65-110).

#### **Appendix D. Robustness of the estimation results.**

A number of robustness checks have been carried out, in order to verify the stability of the above findings. First of all we assessed the stability of the VAR estimation with respect to the cross sectional variability of the annuity premia. Quite remarkably the aggregate longevity risk indicator remains a significant predictor also using maximum (coeff 0.373 t-stat 2.148) and minimum (coeff 0.187 t-stat 1.697) annuity premia rather than the mean ones.

In addition, the VAR(1) has been re-estimated over sub-samples. Truncation of the sample before 2007 does not affect the results, proving that the above findings are not driven by the illiquidity effects induced by the crises. Optimal allocations, the term structure of risk and the risk-return tradeoff discussed above are robust to changes (reductions) of the sample used for the extended VAR(1). Variability is mainly driven by interest rates levels and stock index performances. The sample is essentially characterized by two regimes with a transition around early 90's. The longevity predictor is significant in every estimation on subsamples whose truncation is posterior to 1995. Prior to this period, the estimation indicates that the variability of annuity prices is mainly driven by shocks to short term real and nominal interest rates that are close to being permanent. In fact high (low) interest rate levels imply a reduction (amplification) of the impact of cash flow risk on the price of annuities. As a consequence, diversification benefits deriving from longevity-linked investment are more pronounced in the post-2001 period.

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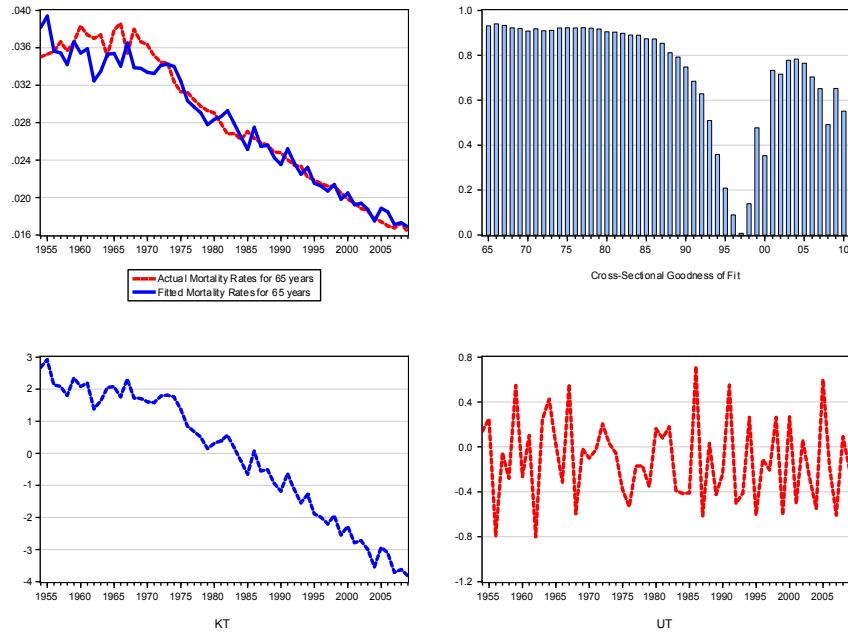


Figure A.1: Lee-Carter Fitted mortality. Figure A.1.1: Fitted mortality rates at 65. Figure A.1.2: Cross-Sectional  $R^2$  of Equation (A.1) Figure A.1.3: The unobservable mortality index  $k_t$ . Figure A.1.4: Innovations in  $k_t$ .

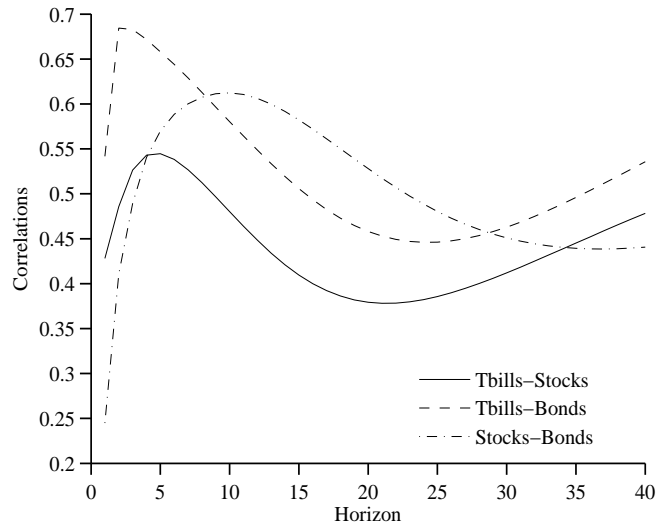


Figure B.1: Term Structure of return correlations between financial securities included in the Extended VAR model

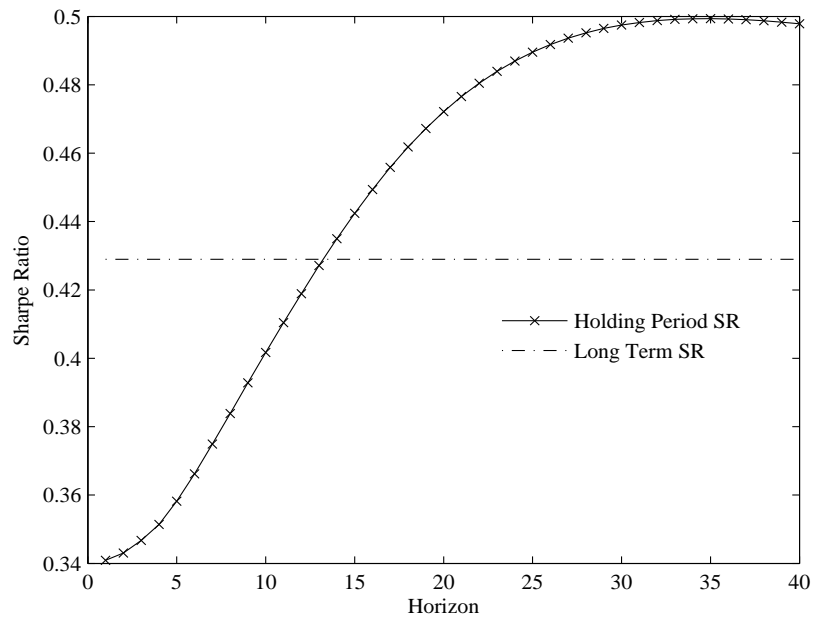


Figure B.2: Term Structure of the Equity Sharpe ratios where the initial level of the state variables is set to their long term expected value.



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	$rtb_t$	$xr_t$	$xb_t$	$y_t$	$(d-p)_t$	$spr_t$	$R^2$	$adjR^2$
	( $t$ )	( $t$ )	( $t$ )	( $t$ )	( $t$ )	( $t$ )		
$rtb_{t+1}$	0.567 (3.825)	-0.027 (-0.662)	-0.110 (-1.843)	0.414 (2.350)	-0.008 (-0.462)	-0.640 (-1.507)	0.530	0.471
$xr_{t+1}$	0.099 (0.192)	0.021 (0.130)	-0.003 (-0.013)	-0.935 (-1.056)	0.107 (1.963)	0.271 (0.171)	0.087	-0.027
$xb_{t+1}$	0.830 (3.649)	-0.087 (-1.286)	-0.508 (-4.315)	0.246 (0.558)	-0.036 (-1.235)	4.684 (5.132)	0.546	0.489
$y_{t+1}$	-0.218 (-3.686)	0.036 (2.463)	0.043 (2.053)	1.006 (9.821)	0.005 (1.337)	0.211 (1.214)	0.784	0.757
$(d-p)_{t+1}$	-0.548 (-0.993)	0.071 (0.402)	0.049 (0.237)	0.398 (0.417)	0.939 (16.822)	1.414 (0.787)	0.872	0.855
$spr_{t+1}$	0.126 (2.778)	-0.030 (-2.680)	0.009 (0.706)	-0.039 (-0.532)	-0.001 (-0.434)	0.405 (3.204)	0.455	0.387

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Cross-Correlations of Residuals

	$rtb$	$xr$	$xb$	$y$	$(d-p)$	$spr$
$rtb$	3.552	0.464	0.070	-0.247	-0.501	0.285
$xr$	-	13.507	0.170	-0.406	-0.977	0.435
$xb$	-	-	7.273	-0.637	-0.191	0.160
$y$	-	-	-	1.403	0.425	-0.845
$(d-p)$	-	-	-	-	14.480	-0.436
$spr$	-	-	-	-	-	1.066

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**VAR(1) - Matrix  $\Phi_1$  - Yearly Sample 1953-2010. Original Financial Variables**

Table B.1: VAR(1) coefficients with relative  $t$ -statistics and Cross-Correlations of Residuals.

**Note:**  $rtb_t$  = ex post real T-bill rate,  $xr_t$  = excess stock return,  $xb_t$  = excess bond return,  $(d-p)_t$  = log dividend-price ratio,  $y_t$  = nominal T-bill yield,  $spr_t$  = yield spread.